It is clear from Fig. 1 that the calculated values of the friction coefficient are in satisfactory agreement with the experimental data.

If the blown gas and the main-stream gas are the same, the equation for the coefficient of friction simplifies to

$$c_f/c_{f_0} = \exp(-\alpha/2). \quad (4)$$

Using Eq. (4), from [1] we can obtain the following expression for the Stanton number St:

$$\frac{\mathrm{St}}{\mathrm{St}_0} = \beta \exp\left(\frac{1-\mathrm{Pr}}{1+\mathrm{Pr}}\,\beta\right) \Big/ \left[1 + \frac{2}{1+\mathrm{Pr}}\,\beta \exp\left(\frac{\beta}{1+\mathrm{Pr}}\right) - \exp\left(\frac{1-\mathrm{Pr}}{1+\mathrm{Pr}}\,\beta\right)\right]. \quad (5)$$

It is clear from Fig. 2 that there is good agreement between the calculated values of St (at Pr = 0.72) and the experimental data up to large values of the blowing parameter β.

NOTATION

c_f is the coefficient of friction; c_{f_0} is the coefficient of friction on impermeable surface;

$$\alpha = \frac{(\rho v)_w}{\rho_\infty U}\,\frac{2}{c_{f_0}}, \quad \beta = \frac{(\rho v)_w}{\rho_\infty U}\,\frac{1}{\mathrm{St}_0}$$

are the blowing parameters; $(\rho v)_w$ is the injected gas flow; ρ_∞, U are the density and velocity in the core flow; k is a coefficient; St_0 is the Stanton number on the impermeable surface; Pr is the Prandtl number; m is the molecular weight. The subscripts 1, 2 relate to the blown gas and the main-stream gas, respectively.

REFERENCES

1. V. D. Sovershennyi, Izv. AN SSSR, Mekhanika zhidkosti i gaza [Fluid Dynamics], no. 3, 1966.
2. C. C. Pappas and A. P. Okuno, JA/SS, 27, no. 5, 1960.
3. L. Green and K. L. Nall, JA/SS, 26, no. 11, 1959.

25 August 1966

EFFECTIVE BOUNDARY CONDITIONS IN STATIONARY HEAT CONDUCTION PROBLEMS

V. S. Ravin

Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 4, pp. 540-541, 1967

UDC 536.2.01

In a number of cases problems of nonstationary heat conduction relating to heat propagation in layered media may be substantially simplified by introducing approximately effective boundary conditions at the interfaces. These approximations are usually based on the low heat capacity of one or more of the media involved in the problem [1, 2].

A similar simplification can also be achieved in stationary problems when in certain media the variation of the temperature field is small in certain directions.* In particular, if at the surface of a massive solid there is a perfect thermal contact with a thin shell of another material not containing heat sources and the boundary conditions are given at the outer surface of the shell, then it is sometimes possible to introduce approximately effective boundary conditions directly at the surface of the massive solid.

1. A body is bounded by the plane z = 0 on which there is a shell bounded by surfaces z = a, F(x, y) = 0. The boundary conditions have the form

$$\left(\alpha \frac{\partial u}{\partial z} + \beta u\right)_{z=a} = g(x, y), \quad u|_{F(x,y)=0} = \omega(z), \quad (1)$$

the thermal conductivities of the body and the shell being, respectively, K and K_1. Starting from the expansion of the temperature function u for $0 \le z \le a$ in a Maclaurin series in z, we obtain at the surface z = 0 the boundary condition

$$\left(\alpha' \frac{\partial u}{\partial z} + \beta u\right)_{z=0} = g(x, y) + R_1, \quad \alpha' = \frac{K}{K_1}(\alpha + \beta a). \quad (2)$$

For the error R_1 we have the estimate

$$|R_1| \leqslant \frac{a}{2}(2\alpha + \beta a)(M_1 + M_2), \quad (3)$$

where

$$M_1 = \max\{\max|\Delta_{xy} u(x, y, 0)|, \max|\Delta_{xy} u(x, y, a)|\},$$

$$M_2 = \max\left|\frac{d^2\omega}{dz^2}\right|, \quad \Delta_{xy} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

2. The small region S_0 of the surface z = 0 of a body is covered with a thin shell bounded by an outer surface S at which the boundary condition is given in the form

$$\left(\alpha \frac{\partial u}{\partial n} + \beta u\right)_S = g(x, y), \quad (4)$$

where n denotes the exterior normal to S.

Averaging the temperature function over the region occupied by the shell and starting from the condition of heat flow balance in the shell we obtain on S_0 the boundary condition

$$\left(\alpha' \frac{\partial u}{\partial z} + \beta u\right)_{z=0} = \bar{g} + R_2, \quad \alpha' = \frac{K}{K_1}\,\frac{S_0}{S}\,\alpha, \quad (5)$$

and for R_2

*Pointed out by G. A. Grinberg.

$$|R_2| \leqslant |\alpha' q_m| + |\beta M|. \quad (6)$$

Here

$$M = \max(u|_{S_0} - \bar{u}|_S), \quad q_m = \max\left(\frac{\partial u}{\partial z} - \overline{\frac{\partial u}{\partial z}}\right)_{S_0},$$

$\bar{g}$, $\bar{u}|_S$, $\overline{\frac{\partial u}{\partial z}}$ are the mean values of g, u on S, $\frac{\partial u}{\partial z}$ on S_0.

REFERENCES

1. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids [Russian translation], Nauka, Moscow, 1964.
2. E. N. Fox, Phil. mag., 18, 209, 1934.

16 July 1966